Sample Paper 7: Paper 2

Question 9 (65 marks)

Question 9 (a) (i)

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$
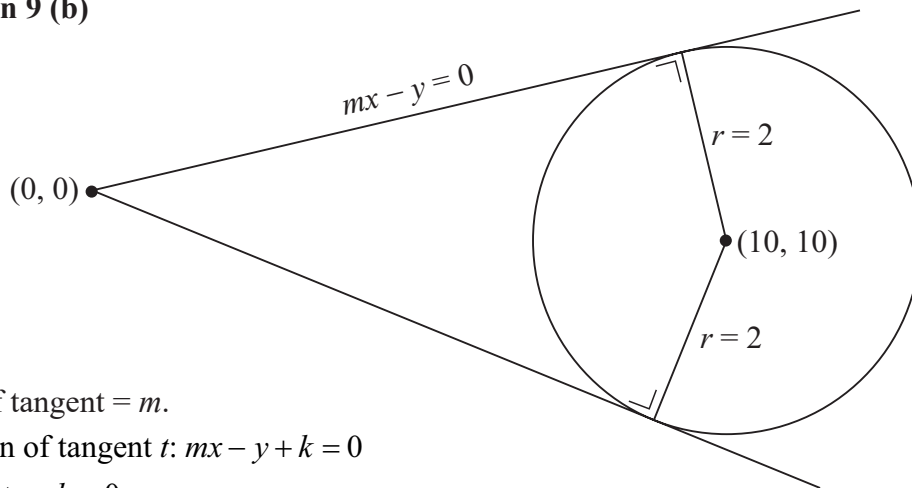
Question 9 (a) (ii)

Equation of circle:

$$x^2 + y^2 - 20x - 20y + 196 = 0$$

$$\text{Centre} = (-g, -f) = (10, 10)$$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{100 + 100 - 196} = \sqrt{4} = 2$$

Question 9 (b)

Slope of tangent = m.

Equation of tangent t: $mx - y + k = 0$

$(0, 0) \in t \Rightarrow k = 0$

Equation of tangent t: $mx - y = 0$

$mx - y = 0$, $(x_1, y_1) = (10, 10)$, $d = 2$

$$2 = \frac{|m(10) - (10)|}{\sqrt{m^2 + 1}}$$

$$2\sqrt{m^2 + 1} = 10|m - 1|$$

$$\sqrt{m^2 + 1} = 5|m - 1|$$

$$m^2 + 1 = 25(m - 1)^2$$

$$m^2 + 1 = 25m^2 - 50m + 25$$

$$24m^2 - 50m + 24 = 0$$

$$12m^2 - 25m + 12 = 0$$

$$(4m - 3)(3m - 4) = 0$$

$$\therefore m = \tfrac{3}{4},\ \tfrac{4}{3}$$

Question 9 (c)

$$x = 10\sqrt{2} \cos 45° = 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 10 \text{ km}$$

$$y = 10\sqrt{2} \sin 45° = 10\sqrt{2} \times \frac{1}{\sqrt{2}} = 10 \text{ km}$$

$$\therefore M(10, 10)$$

Question 9 (d)

The ships travelling from O are moving along tangents to the circle with centre $M(10, 10)$ of radius 2 km. This is analogous to the previous circle. You know the slopes of these two tangents.

Bearing of Ships A and B:

$\tan \alpha = m = \tfrac{3}{4}$

$\therefore \alpha = \tan^{-1}(\tfrac{3}{4}) = 36.87°$ N of E

$\tan \beta = m = \tfrac{4}{3}$

$\therefore \beta = \tan^{-1}(\tfrac{4}{3}) = 53.13°$ N of E

Question 9 (e)

Distances travelled by A and B:

$$v = \frac{s}{t} \Rightarrow s = v \times t$$

$v_A = 40$ km/h, $t = 2$ h $v_B = 30$ km/h, $t = 2$ h

$s = 40 \times 2 = 80$ km $s = 30 \times 2 = 60$ km

Question 9 (f)

$m_1 = \frac{4}{3}$, $m_2 = \frac{3}{4}$

$$\therefore \tan\theta = +\left(\frac{\frac{4}{3} - \frac{3}{4}}{1 + (\frac{4}{3})(\frac{3}{4})}\right) = +\left(\frac{\frac{4}{3} - \frac{3}{4}}{1 + 1}\right) = +\left(\frac{\frac{4}{3} - \frac{3}{4}}{2}\right)$$
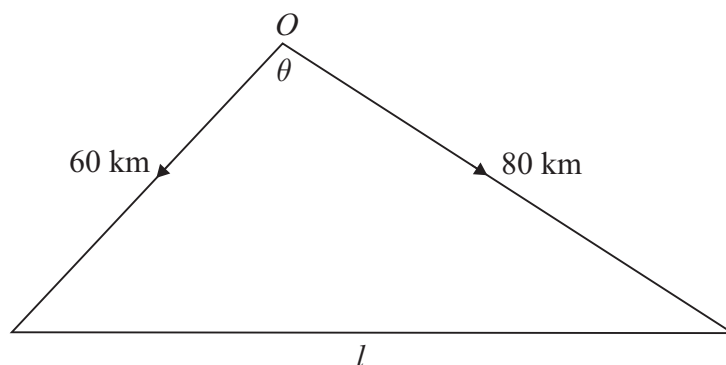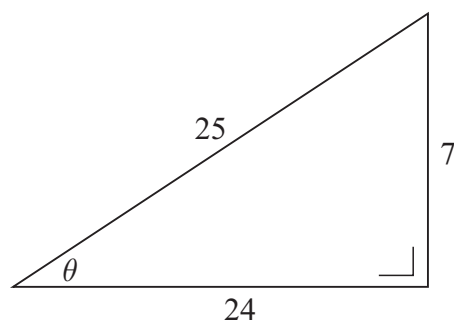
$$= \left(\frac{\frac{4}{3} - \frac{3}{4}}{2}\right) \times \frac{12}{12}$$

$$= \left(\frac{16 - 9}{24}\right)$$

$$= \frac{7}{24}$$

Question 9 (g)

$$\tan\theta = \frac{7}{24} \Rightarrow \cos\theta = \frac{24}{25}$$

$$l^2 = 60^2 + 80^2 - 2(60)(80)\cos\theta$$

$$l = \sqrt{60^2 + 80^2 - 2(60)(80)(\tfrac{24}{25})} = 28 \text{ km}$$